## The NUBASE2012 evaluation of nuclear properties\*

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**Abstract** This paper presents the NUBASE2012 evaluation that contains the recommended values for nuclear and decay properties of nuclides in their ground and excited isomeric  $(T_{1/2} \ge 100 \text{ ns})$  states. All nuclides for which some experimental information is known are considered. NUBASE2012 covers all up to date experimental data published in primary (journal articles) and secondary (mainly laboratory reports and conference proceedings) references, together with the corresponding bibliographical information. During the development of NUBASE2012, the data available in the "Evaluated Nuclear Structure Data File" (ENSDF) database were consulted, and critically assessed of their validity and completeness. Furthermore, a large amount of new and somewhat older experimental results that were missing in ENSDF were compiled, evaluated and included in NUBASE2012. The atomic mass values were taken from the "Atomic Mass Evaluation" (AME2012, second and third parts of the present issue). In cases where no experimental data were available for a particular nuclide, trends in the behavior of specific properties in neighboring nuclei (TNN) were examined. This approach allowed to estimate, whenever possible, values for a range of properties, and are labeled in NUBASE2012 as "non-experimental" (flagged "#"). Evaluation procedures and policies that were used during the development of this database are presented, together with a detailed table of recommended values and their uncertainties.

AMDC: http://amdc.in2p3.fr/ and http://amdc.impcas.ac.cn/

## 1 Introduction

The NUBASE2012 evaluation responds to the needs expressed by the broader nuclear physics community, from fundamental physics to applied nuclear sciences, for a database that contains values of the main nuclear properties such as masses, excitation energies of isomers, halflives, spins and parities, decay modes and their intensities, for all known nuclei in their ground and excited isomeric states. The information presented in NUBASE2012 represents the fundamental building blocks of the modern nuclear physics, and specifically, of the nuclear structure and nuclear astrophysics research.

The main application of NUBASE2012 is the "Atomic Mass Evaluation" (AME2012, second and third parts of this issue) where it is imperative to have an unambiguous identification of all states involved in a particular decay, reaction or mass-spectrometer measurement. This is the primary reason for which the two evaluations are coupled

together in the present issue, for the second time since the existence of the "Atomic Mass Evaluation".

Furthermore, with the advances of modern massspectrometry techniques (see for example Ref. [1] for a recent review) and the availability of intense stable and rare-isotope beams, a large number of unstable nuclei can be produced in a single experiment in their ground and/or isomeric states, and their masses measured with high precision. Thus, NUBASE2012 can be particularly useful for future mass measurements, where an unambiguous identification of complex mass-spectrometry data would be required.

Applications of this database in astrophysics network calculations and in theoretical studies of nuclear properties, where complete and reliable data for all known nuclei are needed, are also envisioned.

Least, but not last, the evaluated data presented in NUBASE2012 could also be useful for specialists in a

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number of applied nuclear fields, such as reactor engineering and design, fuel manufacture and transport, waste management, material analysis, medical diagnostics and radiotherapy, and anywhere when one needs to access basic information on any nuclide.

The information presented in NUBASE2012 fulfills several user-demanded requirements, namely that it is: a) *complete* – includes all measured quantities and their uncertainties, b) *up-to-date* – results from the most recent publications are included, c) *credible and reliable* – identifies and resolves contradictory results that exist in the scientific literature, as well as in other nuclear physics databases, d) *properly referenced* – provides comprehensive information on the validity of all included data.

Most of the data included in NUBASE2012 are in principle available in two other evaluated databases: the "Evaluated Nuclear Structure Data File" (ENSDF) [2] and the "Atomic Mass Evaluation" (AME2012). Therefore, the demand for NUBASE could be partially fulfilled by combining these two databases into a single, 'horizontal' structure, which exists in AME, but not in ENSDF. Therefore, NUBASE2012 could be considered, at a first level, as a critical compilation of those two evaluations.

During the development of the present version of NUBASE, it was imperative to examine all available literature for several nuclides in order to revise results adopted in ENSDF, and to ensure that the recommended data are presented in a consistent way (*credibility and reliability* requirement). It was also necessary to include all the available experimental data, i.e. not only results that were published recently (*up-to-date* requirement), but also somewhat older data that were missing in ENSDF (*completeness* requirement). This implied that some extra evaluation work was carried out, and the corresponding conclusions are added as remarks in the NUBASE2012 table, and in the discussions below. Complete bibliographical references are given for all added experimental data in Table I (see Section 2.7).

There is no strict literature cut-off date for the results presented in the NUBASE2012 evaluation: all data available to the authors until the material was sent for publication (November 18, 2012) have been included. Results that were not incorporated for special reasons, e.g. the need for a heavy revision of the evaluation at a too late stage of development, are added, whenever possible, in remarks to the relevant data.

Note added in proof: ref. [2012Ku.1] reports a large amount of important information, in particular the identification of a series of N - Z = 42 nuclides between Eu and Yb that we include without any estimated mass, half-life or spin/parity.

The contents of NUBASE2012 are described below,

together with the adopted policies that were used during the development of this database. Section 3 presents the updating procedures, while the electronic distribution and interactive display of NUBASE2012 contents by means of a World Wide Web Java program, and by a stand-alone PC-program are described in Section 4.

The present publication includes and updates all the information given in the previous versions of NUBASE: NUBASE1997 [3], and NUBASE2003 [4].

## 2 Contents of NUBASE2012

The NUBASE2012 evaluation contains recommended values for some of the basic nuclear ground state properties, for 3350 nuclides, derived from all available experimental results, together with some values estimated by extrapolating neighboring ones.

NUBASE2012 also contains data on isomeric states (see Section 2.2 for our current definition). We presently know 1256 nuclides which have one, or more, excited isomers in accordance with our definition. In the very first version, NUBASE1997, we used a limited definition of isomers where only states with half-lives greater than 1 millisecond were considered. In NUBASE2003 we started to extend the definition of isomers to nuclidic species living longer than 100 ns. Now, in NUBASE 2012, we include all isomers that have half-lives beyond 100 ns. We also include the description of those states that are involved in mass measurements and thus found in AME2012, see there, Part I, Section 1.1, p. 1288).

NUBASE2012 also contains data on 186 isobaric analog states (IAS), which have their excitation energies determined either through an "internal relation" and taken from ENSDF, or through an "external relation" and then determined by the AME2012 evaluation.

For each nuclide (A, Z), and for each state (ground or excited isomer), the following properties were compiled and, when necessary, evaluated: mass excess, excitation energy of excited isomeric states, half-life, spin and parity, decay modes and their intensities, isotopic abundance (for a stable nuclide), year of discovery and the corresponding bibliographical information for all experimental values of the above items.

References to published articles in the description sections below are given by means of the keynumber style used in the "Nuclear Science Reference" (NSR) bibliographical database [5]. However, references quoted in the NUBASE2012 tables are abbreviated with the first two digits of the year of publication being omitted from the NSR keynumbers. The complete reference list is given at the end of this issue, together with the references used in AME (see AME2012, Part II, p. 1863 in this issue). 104

In NUBASE1997, the names and chemical symbols used for elements 104 to 109 were those recommended by the Commission on Nomenclature of Inorganic Chemistry of the International Union of Pure and Applied Chemistry (IUPAC) [6] at that time. Unfortunately, those names were changed shortly before the publication of NUBASE2003, and two of them were displaced, thus resulting in some confusion (see also AME2012, Part I, Section 6.8, p. 1317). Therefore, the users should be careful when comparing results between NUBASE1997, NUBASE2003 and the present NUBASE2012 evaluations for nuclides with  $Z \ge 104$ . The final adopted names and symbols, as recommended by IUPAC are:

 $(\mathbf{D}\mathbf{f})$ 

104	Tumeriorunum	(KI),
105	dubnium	(Db),
106	seaborgium	(Sg),
107	bohrium	(Bh),
108	hassium	(Hs),
109	meitnerium	(Mt),
110	darmstadtium	(Ds),
111	roentgenium	(Rg),
112	copernicium	(Cn),
114	flerovium	(Fl), and
116	livermorium	(Lv).

muth anfandines

The provisional symbols Ed, Ef, Eh, and Ei are used in NUBASE2012 for yet unnamed elements with Z = 113, 115, 117, and 118, respectively.

NUBASE2012 contains numerical and bibliographical data for all known nuclides for which at least one property is known experimentally, in their ground state, excited isomeric states with  $T_{1/2} \ge 100$  ns, and isobaric analog states. However, it also includes information on, as yet, unobserved nuclides. These were estimated from the observed trends in experimental data of neighboring nuclei (TNN), in order to ensure continuity in the set of considered nuclides simultaneously in *N*, in *Z*, in *A* and in N - Z. The chart of nuclides defined in this way has a smooth contour.

For the yet experimentally unknown properties, values were estimated, here also, from the observed trends in experimental data of neighboring nuclei (TNN). Similarly to AME2012, where masses estimated from trends in the mass surface (TMS) are flagged with the symbol '#', the same symbol is used in NUBASE2012 to indicate TNN non-experimental information.

Such an approach allowed to follow the behavior of a particular nuclear property as a function of N and Z in a consistent way, and it proved beneficial in deducing values for other relevant properties. For example, the excitation energy of the  $J^{\pi}=11/2^{-}$  ( $h_{11/2}$ ) isomer in <sup>179</sup>Tl is not

known experimentally. However, from the extrapolation of values known for the same configuration in the neighboring  $^{177}$ Tl<sup>*m*</sup>,  $^{181}$ Tl<sup>\*</sup> and  $^{183}$ Tl<sup>*m*</sup> nuclides, one can estimate  $E_x=825\#(10\#)$  keV for  $^{179}$ Tl<sup>*m*</sup>. This value, together with the known decay properties of the daughter ( $^{175}$ Au<sup>*m*</sup>) and grand-daughter ( $^{171}$ Ir<sup>*m*</sup>) nuclides, allowed to obtain estimates of the excitation energies of similar isomers in the latter nuclides at  $E_x=167\#(12\#)$  and 160#(14#) keV, respectively, which are not directly measured yet.

As a rule, one standard deviations  $(1 \sigma)$  are used in NUBASE2012 to represent the uncertainties associated with the quoted experimental values. Unfortunately, authors of research articles do not always define the meaning of their reported uncertainties and, under such circumstances, those values were assumed to be one standard deviations. In many cases, uncertainties are not even given at all. They are estimated by us, considering the limitations of the experimental method.

Values and corresponding uncertainties for properties given in NUBASE2012 are rounded off, even if unrounded values were given in the literature or in ENSDF. In cases where the two furthest-left significant digits in the uncertainty were larger than a given limit (set to 30 for the energy levels, and to be consistent with AME2012, and set at 25 for all other quantities, as used in ENSDF), values and uncertainties were rounded off accordingly (see examples in the 'Explanation of table'). In a few cases, that were deemed essential for traceability purposes (e.g. isotopic abundances), the original (unrounded) value is also provided in the associated comment.

#### 2.1 Mass excess

In NUBASE2012 the mass excess values (in keV), defined as being differences between the atomic mass (in mass units) and the mass number, together with their one standard deviation uncertainty, are taken from the mass tables of the AME2012 evaluation (in the third part of this issue, see p. 1608).

In general, knowledge of masses can provide valuable information on decay modes, and in particular on particledecay instability, or beta-delayed particle-decay, for nuclei far from the line of stability. Such information is used in NUBASE2012, and can be seen, for example, in <sup>10</sup>He, <sup>39</sup>Sc, <sup>62</sup>As, or <sup>63</sup>As. In some cases, the claimed observations of decay modes were rejected when it was found that they were not allowed through simple energetics.

Fig. 1 is a complement of the main table, and displays the mass precisions, in a color-coded chart, as a function of N and Z.



#### 2.2 Isomers

In the very first version of NUBASE in 1997 [3], a simple definition for excited isomers, as being excited states with a half-life longer than 1 millisecond, was adopted. All  $\beta$ -decaying states were included in this category, since they have a lower half-life limit of ~1 millisecond (the shortest-lived known  $\beta$ -decaying nuclide is <sup>35</sup>Na with a half-life of 1.5 millisecond). However, already at that time, it was noticed that such a definition had several drawbacks, particularly for neutron-deficient alpha- and proton-decaying nuclides, where much shorter-lived states were known to exist. Moreover, several cases are known where isomers with half-lives far below 1 millisecond survive longer than the ground state itself (e.g. <sup>216</sup>Fr).

In NUBASE2003, the definition of isomers was extended to include excited states with half-lives longer than 100 ns, and in this new edition all such states are now included. The main reasons for this change were to include: a) all shorter-lived proton- and alpha-decaying states observed in many neutron-deficient nuclei,

b) short-lived isomers that may be detected in massspectrometry experiments performed at accelerator facilities following the immediate detection of the produced nuclei, and

c) all possible isomers that may be detected in such experiments in the future.

In NUBASE2012, isomers are given in order of increasing excitation energy, and identified by appending the letters 'm', 'n', 'p', 'q', or 'r' to the nuclide name, e.g. <sup>90</sup>Nb for the ground state, <sup>90</sup>Nb<sup>m</sup> for the first excited isomer, <sup>90</sup>Nb<sup>n</sup> for the second one, and <sup>90</sup>Nb<sup>p</sup>, <sup>90</sup>Nb<sup>q</sup>, and <sup>90</sup>Nb<sup>r</sup> for the third, fourth and fifth ones, respectively. In only two instances, <sup>179</sup>Ta and <sup>214</sup>Ra, a sixth isomer had to be included. They were labeled provisionally with the letter 'x'.

The excitation energy of a given isomer can be determined using different experimental methods, which, in general, belong to the category of either internal or external relations. A typical internal relation is via the  $\gamma$ -ray decay energy, or a combination of such  $\gamma$ -ray energies. The most accurate values for the excitation energies of isomers that are deduced by this approach can be found in ENSDF, where a least-squares fitting procedure is applied to all  $\gamma$ rays along the decay path of a particular isomer. However, when no such internal relations can be established, then the relation to other nuclides (external relations) can be used to deduce the mass (or energy) difference between excited and ground state isomers. In all such cases, the most accurate values can only be derived using the AME evaluation procedure. The values are therefore taken from AME2012. The origin (the method used to establish the external relation) of each isomer data element is then indicated by a two-letter code, next to the isomer excitation energy, in the NUBASE2012 table (see the Explanation of Table I, p. 1176). For internal relations, the origin field is left blank and the numerical values are taken either from ENSDF or from literature updates. In the latter case, a least-squares fit to the measured  $\gamma$ -ray decay energies from complex level schemes was applied, in accordance with the current ENSDF policies.

An interesting example is the proton decay of <sup>167</sup>Ir, studied using a parent-daughter correlation technique [1997Da07], where the excitation energy of the isomer was determined as  $E_x = 175.3(2.2)$  keV. This information is displayed by the 'p' symbol in the origin field. In addition, recent  $\alpha$ -decay studies of proton-rich nuclides using a spatial and time correlation technique with highly-segmented silicon strip detectors not only showed that a number of  $\alpha$  lines assigned earlier to ground states belong in reality to isomers, but they also determined values for their excitation energies.

Another example is <sup>181</sup>Pb, where the  $\alpha$ -decay halflife that was previously assigned to <sup>181</sup>Pb<sup>m</sup> is now associated with the ground state [1996To01]. More recent work [2005Ca.A, 2005Ca43,2009An20] established that the main  $\alpha$  line feeds the 77 keV excited state in <sup>177</sup>Hg, and subsequently decays to the ground state via  $\gamma$ -ray emission.

It also happens that connections between excited and ground state isomers can be obtained by both internal relations and one, or more, external relations with comparable accuracies. All relations are then combined within the AME2012 data by adding an equation that relates the excitation energy obtained from ENSDF (or from literature), so that the AME2012 derives the best combination of all data. For example, the AME2012 derives the mass of <sup>178</sup>Lu<sup>m</sup> at 66% from  $E_x(IT)=120(3)$  keV [1993Bu02] and at 34% from <sup>176</sup>Lu(t,p)<sup>178</sup>Lu<sup>m</sup>=4482(5) keV [1981Gi01]. The adjusted excitation energy is thus 123.8(2.6) keV.

In some cases excitation energies known from internal relations are essential in order to determine the mass of the ground state. Those values are labeled in the NUBASE table with 'IT' in the origin field. They are entered as an equation in AME2012 so that the ground state mass can be derived (e.g. <sup>72</sup>Zn<sup>m</sup>, <sup>197</sup>Pb<sup>m</sup>, or <sup>234</sup>Pa<sup>m</sup>). Similarly, when the precision of the excitation energy is not much higher than that deduced from the energy relation to another nuclide, the internal relation is then added to the AME2012 dataset, in order to use this accuracy (e.g. <sup>181</sup>Hg<sup>m</sup> or <sup>185</sup>Tl<sup>m</sup>). An interesting case is the mass and excitation energy of the second excited isomer of <sup>186</sup>Tl (<sup>186</sup>Tl<sup>n</sup>), where only its mass is experimentally known from a Penning trap (ISOLTRAP) measurement [2012Bo.A]. However, the well

known transition from <sup>186</sup>Tl<sup>*n*</sup> to <sup>186</sup>Tl<sup>*m*</sup> allows to determine not only the mass of the latter isomer, but also the excitation energy of the  $\alpha$ -decaying isomer in the parent nuclide, <sup>190</sup>Bi<sup>*m*</sup>.

When the existence of an isomer is under discussion, it is flagged with 'EU' ("existence uncertain") in the origin field (e.g.  $^{73}$ Zn<sup>*n*</sup>). A comment is generally added to indicate why this existence is questioned, or where this matter is discussed in more detail. Depending on the degree of confidence in the existence of a particular state, the mass excess and excitation energy values can be given or omitted in the NUBASE table (e.g.  $^{138}$ Pm<sup>*n*</sup>). In the latter case, the label "non existent" appears in the place of the excitation energy field.

When a particular isomer was initially reported as "discovered", but later it was proved to be an error, it is flagged with 'RN' in the origin field, indicating "reported, non existent" (e.g.  $^{248}\text{Es}^m$ ). In such cases, no mass excess or excitation energy values are given, and, similarly to the 'EU' choice above, a "non existent" label is added.

*Note*: the use of the two flags, 'EU' and 'RN', was extended to cases where the discovery of a nuclide is questioned (e.g. <sup>260</sup>Fm or <sup>289</sup>Lv). However, an estimate for the ground state mass, derived from trends in the mass surface (TMS), is always given in AME2012 and NUBASE2012.

In several instances, lower and higher limits for the excitation energy of a particular isomer are presented in ENSDF. The policy of NUBASE2012 is that a uniform distribution of probabilities is assumed, which yields a mid-range value and a  $1\sigma$  uncertainty correspondint to 29% of the range (see Appendix B of the AME2012, Part I, p. 1326, for a complete description of this procedure). For example, the excitation energy of the  ${}^{162}\text{Tm}^m$ isomer is known from ENSDF to be above the 66.90 keV level. However, there is also solid experimental evidence that it is below the 192 keV level, and so this information is presented (after rounding off) as  $E_x = 130(40)$  keV in NUBASE2012. When such a value is based on theoretical considerations, or from TNN, the resulting  $E_x$  is considered as a non-experimental quantity and the value is consequently flagged with the '#' symbol.

In cases where the uncertainty of the excitation energy,  $\sigma$ , is relatively large as compared to the  $E_x$  value, the assignment of the level as a ground or isomeric state is uncertain. If  $\sigma > E_x/2$ , a '\*' flag is added in the NUBASE2012 table.

The ordering of several ground and excited isomeric states were reversed as compared to the recommendations in ENSDF. These cases are flagged with the '&' symbol in the NUBASE2012 table. In several other instances, evidence was found for states located below the adopted ground state in ENSDF. There are also cases where the

trends in neighboring nuclides, with the same parities in N and Z, strongly suggest that such a lower state should exist. Such results were added in the NUBASE2012 table and are easily located, as they are flagged with the '&' symbol. In a growing number of cases, new experimental information on masses led to a reversal of the ordering between previously assigned ground and excited isomeric states.

Thanks to the coupling of the NUBASE2012 and AME2012 evaluations, all changes in the ordering of nuclear levels have been carefully synchronized.

Finally, there are cases where data exist on the order of the isomers, e.g. if one of them is known to decay into the other one, or if the Gallagher-Moszkowski rule [7] for relative positions of combinations points strongly to one of the two as being the ground-state. Detailed discussions can be found in Ref. [8]. Only for  $^{256}$ Md, we discovered too late, relative to the publication deadline, that the adopted ordering had to be reversed, we only added a remark to  $^{256}$ Md<sup>m</sup> (see p. 1278).

#### 2.2.a Isobaric analog states (IAS)

The recent revived interest in isomeric states has naturally led to taking a closer look at the historically very popular subject of isobaric analog states (see AME2012, Part I, Section 6.4, p. 1314). In NUBASE2012 we have included mainly the T = 3/2 to T = 3 experimentally observed IAS. These states are generally labelled with *i* or *j* superscripts, for members of successively higher multiplets.

Some nuclides belong simultaneously to several categories, for example, they may be in their ground state but they may also be the IAS of some other ground state nucleus, as is the general case for ground state mirror nuclei. Here, the IAS label is not present since these nuclides are already naturally included in the database. Another exception is the set of N = Z T = 1 odd-odd ground state nuclides, accessible via super-allowed beta decay, and which are also already part of the original dataset of ground state masses. They are: <sup>34</sup>Cl<sub>17</sub>, <sup>42</sup>Sc<sub>21</sub>, <sup>46</sup>V<sub>23</sub>, <sup>50</sup>Mn<sub>25</sub>, <sup>54</sup>Co<sub>27</sub>,  $^{62}\text{Ga}_{31}$  and  $^{70}\text{Br}_{35}.$  The reader might note that only the Z = 29 and Z = 33 do not show up in this series, since their ground states are T = 0, as expected from theory. Finally, there are eight excited isomers, <sup>16</sup>N<sup>m</sup>, <sup>26</sup>Al<sup>m</sup>, <sup>34</sup>Cl<sup>m</sup>, <sup>38</sup>K<sup>m</sup>,  ${}^{46}V^m$ ,  ${}^{50}Mn^m$ ,  ${}^{54}Co^m$  and  ${}^{72}Ga^m$  which are also IAS. In such cases, the isomer labels ('m', 'n',...) are used preferentially over the IAS labels. Here we note with interest that five of them have experimental excitation energies determined, at least partly, by the JYFLTRAP-Jyväskylä Penning trap (see AME2012, Part I, Section 6.2, p. 1311).

In NUBASE2012 there are roughly 180 unique IAS

masses, of which 107 are evaluated in the AME via external relations, and 70 cases previously evaluated through internal relations and published in ENSDF. There are a remaining seven cases where no clear experimental data is available, and although some Isobaric Multiplet Mass Equation (IMME) [9] and Coulomb Displacement Energy (CDE) [10] calculations point to a likely IAS state, their existence cannot yet be certified experimentally (see for example <sup>56</sup>Zn<sup>*i*</sup> and the discussion in [2007Do17]).

The isospin multiplet appartenance given in the table is the logical IAS multiplet value, and has not necessarily been deduced experimentally.

#### 2.3 Half-life

For some light nuclei, the half-life  $(T_{1/2})$  is deduced from the total level width  $(\Gamma_{\rm cm})$  by the equation  $\Gamma_{\rm cm} T_{1/2} \simeq \hbar \times \ln 2$ :

$$T_{1/2}$$
 (s)  $\simeq 4.562 \, 10^{-22} / \Gamma_{\rm cm} \, ({\rm MeV}).$ 

The following units are used for a convenient display: seconds (s) and its sub-units, minutes (m), hours (h), days (d) and years (y) and its sub-units. Conversion between years and seconds or days could follow various definitions: Julian year, Gregorian year, tropical year 1900, epoch 2000,..., differing only slightly from each other. A fixed value of:

1 y = 31 556 926 s or 1 y = 365.2422 d

was adopted in NUBASE2012.

Asymmetric uncertainties for half-lives,  $T_{1/2}{}^{+a}_{-b}$ , are often presented in the literature. However, in order for these values to be used in practical applications, they need to be symmetrized. A rough symmetrization procedure was used earlier (see AME1995) where the central value was taken as the mid-value between the upper and lower  $1\sigma$ -equivalent limits,  $T_{1/2} + (a-b)/2$ , and the uncertainty was defined to be the average of the two uncertainties, (a+b)/2. A strict statistical derivation (see Appendix A) shows that a better approximation for the central value can be obtained by using

 $T_{1/2} + 0.64 \times (a - b).$ 

The exact expression for asymmetric uncertainties, adopted in NUBASE2012, is presented in Appendix A.

When two or more independent measurements were reported in the literature, the corresponding values were weighted by their reported precisions and then averaged. While doing this, the NORMALIZED CHI,  $\chi_n$  (or 'consistency factor' or 'Birge ratio'), as defined in AME2012, Part I, Section 5.2, p. 1306) is considered. Only when  $\chi_n$  is larger than 2.5, departure from the statistical result is allowed, and the external uncertainty for the average result is adopted. This follows the same policy that is discussed and adopted in AME2012, Part I, Section 5.4, p. 1307. Very rarely, when  $\chi_n$  is so large that all individual uncertainties can be considered as irrelevant, the arithmetic (unweighed) average is adopted and the corresponding uncertainty is based on the dispersion of the values. In such cases, the list of values that were averaged, together with the  $\chi_n$  value (when relevant) and the reason for this choice, are given in the NUBASE2012 table. When contradictory (conflicting) results were identified in the literature, a great deal of attention was focused on establishing the reason for such discrepancies, and consequently, to reject the corresponding bad data. The reasons for such decisions are given as comments in the NUBASE2012 table.

In experiments where extremely rare events are detected and where the results are very asymmetric (e.g. studies of super-heavy nuclei), the half-life values reported in different publications were not directly averaged. Instead, when the information presented in the literature was sufficient (e.g. <sup>264</sup>Hs or <sup>269</sup>Hs), the delay times associated with the individual events were combined, as prescribed by Schmidt *et. al.* [1984Sc13].

Some experimental results are reported in the literature as a range of values with a most probable lower and upper limit. These are treated, as in the case of isomer excitation energies (see preceding page), as a uniform distribution of probabilities.

In the NUBASE2012 table, an upper or a lower limit on the half-life value is given for nuclides identified using a time-of-flight technique. The following policies were considered:

i) For *observed* nuclides, the lower limit for the half-life is given in place of the uncertainty (see <sup>44</sup>Si, p. 1188). However, such limits should be used with caution, since it may be far below the actual half-life. In order to avoid confusion, a somewhat more realistic estimate (flagged with #), derived from trends in the half-life values of neighboring nuclides (TNN), is also given.

ii) For nuclides that were sought for, but *not observed*, the upper limit is given in place of the actual half-life uncertainty. Upper limits for a dozen of undetected nuclides were evaluated by F. Pougheon [1993Po.A], based on the time-of-flight of the experimental setup and the production yields expected from TNN (e.g. <sup>21</sup>Al).

When ground state half-lives for nuclides with the same parities in Z and N are found to vary smoothly (see Fig. 2), interpolation or extrapolation procedure is used to obtain reasonable estimates for unknown nuclei.















Similarly to ENSDF, spin and parity values are presented with and without parentheses, based on strong and weak assignment arguments, respectively (see the introductory pages of Ref. [11]). Unfortunately, the latter include estimates from theory or from TNN (trends in neighboring nuclides). In NUBASE2012, following our policy of making a clear distinction between experimental and non-experimental information, parentheses are used if the so-called "weak" argument is based on experimental observations, while the symbol '#' is used for the other cases. It should also be noted that despite the well defined evaluation policies [11], there are a number of inconsistencies in ENSDF regarding the spins and parities for excited states. Often, proposed assignments reflect the interpretation of a particular ENSDF evaluator, rather than that of firm policy rules. As a result, assignments to similar states in neighboring nuclei are put in parenthesis by one evaluator, but not by other, although similar experimental information is available. We have tried to use a consistent approach in assigning spins and parities to excited states, but the survey is still far from complete and the reader may still find inconsistencies. The authors would gratefully appreciate feedback from users on such cases, in order to improve future versions of NUBASE.

If spins and parities are not determined experimentally, they can be estimated from trends in neighboring nuclides (TNN) with the same parities in N and Z. Although, this is frequently the case for odd-A nuclides (see Fig. 3 and Fig. 4), such trends are also sometimes valid for odd–odd isotopes, especially in the neighborhood of magic numbers, as can be seen in Fig. 5. In all cases, the estimated values are flagged with the '#' symbol.

The review of nuclear radii and moments of Otten [1989Ot.A], in which the spins were compiled, was used to check and complete the spin values in NUBASE2012.

*Note added in proof*: A novel technique based on a condensed-matter device allows to determine absolute values for the spin [2012Vi.1], similarly to the hyperfine method. This technique was applied to <sup>159</sup>Tb and confirmed the well known spin of 3/2.

#### 2.5 Decay modes and their intensities

The most important policy in assembling the information for the decay modes was to establish a very clear distinction between a decay mode that is energetically allowed, but not experimentally observed yet (represented by a question mark alone, which thus refers to the decay mode itself), and a decay mode which is actually observed, but for which the intensity could not be determined (represented by '=?', the question mark referring here to the quantity after the equal sign).

Similarly to ENSDF, no corrections were made to normalize the primary intensities to 100%.

In addition to applying direct updates from the literature, partial evaluations completed by other authors were also considered, which are properly referenced. Those cases are mentioned below, when discussing some particular decay modes.

## $\beta^+$ decay

In the course of this work some definitions and notations for  $\beta^+$  decay were refined, in order to provide a clearer presentation of the available information. Specifically,  $\beta^+$  denotes the decay process that includes both electron capture, labeled  $\varepsilon$ , and decay by positron emission, labeled e<sup>+</sup>. One can then symbolically write:  $\beta^+ = \varepsilon + e^+$ . It is well known that for an available energy below 1022 keV, only electron capture,  $\varepsilon$ , is allowed, whilst above that value the two processes are in competition.

*Remark*: this notation is **not** the same as the one used implicitly in ENSDF, where the combination of both modes is denoted " $\varepsilon + \beta^+$ ".

When both modes compete, the separated intensities are not always experimentally available and frequently they are deduced from model calculations, as is the policy in ENSDF. In continuation of one of the general NUBASE policies, in which only experimental information is used whenever possible, it was decided not to retain the separated values calculated in ENSDF (which are scarce and not always updated). Only in a few very specific cases, where the distinction is of importance, such as in the case of rare or extremely rare processes (e.g. <sup>91</sup>Nb, <sup>54</sup>Mn, <sup>119</sup>Te<sup>m</sup>), separate values are given.

By the same token, both electron capture-delayed fission ( $\varepsilon$ SF) and positron-delayed fission ( $e^+$ SF) are given with the same symbol  $\beta^+$ SF.

#### Double- $\beta$ decay

In the course of this work it was found that half-lives for double  $\beta$ -decaying nuclides were not always consistently given in ENSDF. Since the two-neutrino gs-gs transition is the dominant decay process (one exception may be <sup>98</sup>Mo, for which the neutrinoless decay is predicted to be faster, see [2002Tr04]), only those half-life values or their upper-limits were presented in the NUBASE2012 table. No attempt was made to convert the half-life values to the same statistical confidence level (CL) upper limit results given by different authors.

The excellent compilation of Tretyak and Zdesenko [2002Tr04] was of great help in covering such decays.

#### $\beta$ -delayed particle decays

For delayed particle decays, intensity relations have to be carefully considered. By definition, the intensity of a decay mode is the percentage of decaying parent nuclei in that mode. But traditionally, the intensities of the pure  $\beta$  decay are summed with those of the delayed particles in order to give an intensity that is assigned to the pure  $\beta$  decay. For example, if the (*A*, *Z*) nuclide has a decay described traditionally by ' $\beta^{-}$ =100;  $\beta^{-}$ n=20', this means that for 100 decays of the parent, 80 (*A*, *Z*+1) and 20 (*A*-1, *Z*+1) daughter nuclei are produced and that 100 electrons and 20 delayed neutrons are emitted. A strict notation in this case, using the definition above, would be ' $\beta^{-}$ =80;  $\beta^{-}$ n=20'. However, in the present work, it has been decided to follow the traditional notation, and so we write: ' $\beta^{-}$ =100;  $\beta^{-}$ n=20'.

This also holds for more complex delayed emissions. For example, a decay described by: ' $\beta^{-}=100$ ;  $\beta^{-}n=30$ ;  $\beta^{-}2n=20$ ;  $\beta^{-}\alpha=10$ ' corresponds to the emission of 100 electrons, ( $30+2\times20=70$ ) delayed-neutrons and 10 delayed- $\alpha$  particles; and in terms of residual nuclides, to 40 (A,Z+1), 30 (A-1,Z+1), 20 (A-2,Z+1) and 10 (A-4,Z-1). More generally, the number of emitted neutrons per 100 decays,  $P_n$ , can be written as:

$$P_{\rm n} = \sum_i i \times \beta_{i{\rm n}}^-;$$

and similar expressions can be written for  $\alpha$  and proton emission. The number of residual daughter nuclides (*A*, *Z*+1) populated via  $\beta^-$  decay is then:

$$\beta^- - \sum_i \beta^-_{in} - \sum_j \beta^-_{j\alpha} - \dots$$

Another special remark concerns the intensity of a particular  $\beta$ -delayed mode. In general, the primary (parent)  $\beta$  decay populates several excited states in the daughter nuclide, which can further decay by particle emission. However, in a case where the ground state of the daughter nuclide decays also by the same particle emission, some authors included its decay in the value for the corresponding  $\beta$ -delayed intensity. It has been decided to not use such an approach in NUBASE2012 for two main reasons. First, the energies of delayed particles emitted from excited states are generally much higher than those emitted from the ground state, hence implying different subsequent processes. Secondly, since the characteristic decay times from excited states are related to the parent, whereas decays from the daughter's ground state are connected to the daughter nuclide itself. For example <sup>9</sup>C decays via  $\beta^+$  with an intensity of 100% of which 12% and 11% populate two excited proton-emitting states in <sup>9</sup>B, and 17% goes to an  $\alpha$ -emitting state. Thus,  $\beta^+ p=23\%$ and  $\beta^+\alpha=17\%$ , from which the user of the NUBASE2012 table can derive a 60% direct feeding of the ground state of <sup>9</sup>B. In a slightly different example, <sup>8</sup>B decays to only two excited states in <sup>8</sup>Be, which in turn decay by  $\alpha$ - and  $\gamma$ -ray emissions, but not to the <sup>8</sup>Be ground state. Thus, one may write  $\beta^+=100\%$  and  $\beta^+\alpha=100\%$ , the difference of which leaves 0% for the feeding of the daughter's ground state.

Finally, the users should be aware that the percentages given in the NUBASE2012 table are related to 100 parent decaying nuclei, rather than to the primary beta-decay fraction. An illustrative example is given by the decay of <sup>228</sup>Np, for which the delayed-fission probability is given in the original paper as 0.020(9)% [1994Kr13], but this value is relative to the  $\varepsilon$  process, which has an intensity of 59(7)%. Thus, the renormalized delayed-fission intensity is 0.020(9)% × 0.59(7) = 0.012(6)% of the total decay intensity.

In compiling the data for delayed proton and  $\alpha$  activities, the remarkable work of Hardy and Hagberg [1989Ha.A], in which the corresponding physics was reviewed and discussed in detail, was consulted. The review of Honkanen, Äystö and Eskola [12] on delayed proton decays has also been used.

Similarly, the review of delayed neutron emission by Hansen and Jonson [13] was carefully examined and used in the NUBASE2012 table, together with the evaluation of Rudstam, Aleklett and Sihver [1993Ru01].

#### 2.6 Isotopic abundances

Isotopic abundances are taken from the compilation of M. Berglund and M.E. Wieser [2011Be53] and the values are listed in the decay field with the symbol *IS*. These data are given in the NUBASE2012 table as presented originally in [2011Be53], and so in this case the rounding off policy was not applied.

#### 2.7 References

The year of the archival file for the nuclides evaluated in ENSDF is indicated, otherwise this entry is left blank.

References for all of the experimental updates are given by the NSR keynumber style [5], and are listed at the end of this issue (p. 1863). They are followed by one, two or three one-letter codes which specify the added or modified physical quantities (see the Explanation of Table I, p. 1176). In cases where more than one reference is needed to describe a particular update, they are given as a remark. No reference is given for estimated values. The initials of the present authors, AHW, FGK, GAU, JBL, MMC, WGM, are used as reference keys in cases where it may not be precisely clear that the re-interpretation of data were made by the present authors.

## **3** Updating procedure

In general, NUBASE is updated via two routes: from ENSDF after each new *A*-chain evaluation is published (or from the bi-annual releases), and directly from the literature. Data available in the "Unevaluated Nuclear Data List" (XUNDL) are also regularily consulted [14].

ENSDF files are retrieved from NNDC using the online service [2]. The programs, originally developed by O. Bersillon and one of the present authors (JB) [15], are used to successively:

• check that each Z in the A-chain has an 'adopted levels' data set; if not, a corresponding data set is generated from the 'decay' or 'reaction' data set,

• extract the 'adopted levels' data sets from ENSDF,

• extract the required physical quantities from these data sets, and convert them into the NUBASE format.

The processed data are used to manually update the previous version of NUBASE. This step is repeated independently by two, and sometimes by three, of the present authors, and cross-checked until complete agreement is reached.

ENSDF is updated generally by A-chains and more recently also by individual nuclides. Its contents, however, are very large, since it encompasses all of the complex nuclear structure and decay properties. This is a huge effort, and it is not surprising that occasionally some older data (in particular annual reports, conference proceedings, and theses) are missing, and that some recent data have not yet been included. When such cases were revealed, they were analyzed and evaluated, as described above, and the NUBASE2012 database was updated accordingly. In principle, these new data will be included in future ENSDF evaluations and the corresponding references can then be removed from future NUBASE distributions. Unfortunately, it has been observed in the past that such a procedure was not always adhered to. In fact, in some newer ENSDF files, quotations to earlier NUBASE publications were found, which leads to an indesirable loop resulting in non-traceable information.

#### **4 Distribution and displays of NUBASE2012**

The full contents of the present evaluation is available on-line at the web site of the Atomic Mass Data Center (AMDC) [16]. An electronic ASCII file for the NUBASE2012 table is also distributed by the AMDC website. These files will **not** be updated, to allow stable reference data for various calculations. Any work using those files should make reference to the present paper and not to the electronic files.

The contents of NUBASE2012 can be displayed by a stand-alone PC-program called "NUCLEUS" [17], which

can also be downloaded from the AMDC website. It will be updated on regular basis to allow users to check for the latest available information in NUBASE database.

### 5 Conclusions

The 'horizontal' evaluated database, NUBASE2012, which contains the recommended values for the main properties of all known nuclides in their ground and excited isomeric states, was developed. This has been completed for the first time by the inclusion of all available IAS data. These data originate from a compilation of two evaluated databases: ENSDF, followed by a critical assessment of the validity and completeness of those data, including new updates from the literature, and AME2012. The main requirement in developing NUBASE2012 was to cover as completely as possible all the available experimental data and to provide proper references to them, especially for cases that are not already included in ENSDF. This traceability allows any user to check the recommended data and, if necessary, to undertake a re-evaluation.

As a result of this 'horizontal' work, better homogeneity in handling and presentation of all data was obtained for all known nuclides. Furthermore, isomeric assignments and their excitation energies were reconsidered on a firmer basis and their data improved.

It is expected to follow up this third version of NUBASE with improved treatments in the future. A foreseeable implementation would be to provide the main  $\alpha$ ,  $\gamma$ , conversion electrons and X-ray lines accompanying particular decays, as well as to include even shorter-lived excited nuclear isomers. NUBASE could also be extended to other nuclear properties, such as energies of the first 2<sup>+</sup> states in even-even nuclides, radii, moments, etc.

A new feature that was implemented in the present version of NUBASE is the compilation of the year of discovery for each nuclide in its ground or isomeric state. For the former, recent evaluations performed by a group at Michigan State University [18] were adopted. Similar criteria was used when assigning the year of discovery for isomeric states. However, we would like to make the users aware that this feature for excited isomers is not fully checked and that there are still some cases missing.

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Figure 8: Simulated asymmetric probability density function (heavy solid line) and the equivalent symmetric one (dashed line).

# Appendix A Symmetrization of asymmetric uncertainties

Experimental data are sometimes given with asymmetric uncertainties,  $X_{-b}^{+a}$ . If these data are to be used in some practical applications, their uncertainties may need to be symmetrized. A simple method (Method 1) that was developed earlier, uses the central value to be in the middle between the upper and lower 1  $\sigma$ -equivalent limits

X + (a - b)/2, with the uncertainty defined to be the average of the two uncertainties

$$(a+b)/2.$$

An alternative method (Method 2) considers the random variable *x* associated with the measured quantity. For this random variable, one assumes that the probability density function is an asymmetric normal distribution having a modal (most probable) value of x = X, a standard deviation *b* for x < X, and a standard deviation *a* for x > X(Fig. 8). Then the average value of this distribution is

$$\langle x \rangle = X + \sqrt{2/\pi} (a - b),$$

with variance

$$\sigma^2 = (1 - 2/\pi) (a - b)^2 + ab.$$
(1)

The median value m which divides the distribution into two equal areas is given, for a > b, by

$$\operatorname{erf}\left(\frac{m-X}{\sqrt{2}a}\right) = \frac{a-b}{2a},$$
 (2)

and by a similar expression for b > a.

One can then define the equivalent symmetric normal distribution that have a mean value equal to the median value m of the previous distribution with same variance  $\sigma$ .

If the shift m - X of the central value is small compared to *a* or *b*, expression (2) can be written [19]:

$$m - X \simeq \sqrt{\pi/8} (a - b)$$
$$m - X \simeq 0.6267 (a - b).$$

In order to allow for a small non-linearity that appears for higher values of m - X, the relation

$$m - X = 0.64 \left(a - b\right).$$

was adopted for Method 2.

Table A illustrates the results from both methods. In NUBASE2012, Method 2 is used for the symmetrization of asymmetric half-lives and decay intensities.

Table A. Examples of two different treatments of asymmetric half-life uncertainties.
Method 1 is the classical method, used previously, as in the AME1995.
Method 2 is the one developed in NUBASE2003, described in this Appendix.

Nuclide	Original $T_{1/2}$	Method 1	Method 2	
<sup>54</sup> Zn	3.2+1.8-0.8 ms	$3.7 \pm 1.3$	$3.8 \pm 1.3$	
<sup>80</sup> Cu	170+110-50 ms	$\frac{200 \pm 80}{200 \pm 80}$	$208 \pm 83$	
<sup>83</sup> Mo	6+30–3 ms	$20\pm17$	$23\pm19$	
<sup>100</sup> Kr	7+11–3 ms	$11 \pm 7$	$12\pm 8$	
<sup>115</sup> Mo	51+79–19 ms	$81\pm49$	$89\pm53$	
<sup>222</sup> U	1.0+1.0–0.4 μs	$1.3\pm0.7$	$1.4\pm0.7$	
<sup>264</sup> Hs	327+448–120 μs	$490\pm280$	$540\pm300$	
<sup>265</sup> Rf	105+503–48 s	$332\pm275$	$400\pm320$	
<sup>266</sup> Mt	1.01+0.47-0.24 ms	$1.1\pm0.4$	$1.2\pm0.4$	
<sup>267</sup> Db	73+350–33 m	$230\pm190$	$280\pm220$	

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